



Article

Analytical Method for Determining the Viscosity Index of Engine Lubricating Oils

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Abstract: This paper proposes a simple analytical method for determining the viscosity index (*VI*) that effectively aligns with results obtained from the applicable standards. This method simply needs the kinematic viscosity of the tested oil at 40 and 100 °C as input parameters and does not need to use tables that are an integral part of the mentioned standards. This work presents a method and evaluates the accuracy of determining auxiliary parameters in the form of kinematic viscosity values at 40 °C for two hypothetical oils. These oils have a kinematic viscosity at 100 °C equal to that of the oil under testing and have *VI* = 0 and 100. The relative fitting error percentage and the coefficient of determination are found for the specified auxiliary indicators. The method is validated using data obtained from previous studies in the form of the kinematic viscosity of lubricating oil–diesel mixtures. The mixtures of viscosity grades SAE 30 and SAE 40 lubricating oil with diesel oil at concentrations of 0, 1, 2, 5, 10, 20, and 50% w/w are tested. The viscosity index for each mixture is determined using a standard-based manual calculation using the Anton Paar viscosity index calculator and the proposed method. The results obtained from the proposed analytical method are compared with those from two other methods. The maximum percentage relative fitting error ($\delta_{max} \approx 1\%$) and the coefficient of determination ($R^2 > 0.999$) are determined. The obtained results demonstrate a very good fit and, thus, confirm the usefulness of the proposed approach.



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Keywords: viscosity index; kinematic viscosity; viscosity variation with temperature; lubricating oil; analytical algorithm

1. Introduction

Viscosity is a critical characteristic of fluids (liquids and gases) and plastic solids, representing their internal friction resulting from the movement of fluid layers during flow. Dynamic viscosity and kinematic viscosity are distinguished from each other [1]. Dynamic viscosity η_t (Pa·s) represents the shear stress ratio τ (kg/m) to the shear rate $\dot{\gamma}$ (1/s), while the kinematic viscosity at a given temperature t is the ratio of the dynamic viscosity η_t (Pa·s) to the fluid density ρ_t (kg/m³) [2]. The liquid parameters are analyzed in the subsequent parts of this article.

The viscosity of liquids decreases with increasing temperature t (°C), which is due to an increase in the energy of the liquid particles and the velocity of the liquid layers, thus reducing the contact time of the particles [3]. The shorter interaction time is equivalent to a reduced internal friction resistance. For dynamic viscosity, this change is described by the Arrhenius–Guzman relationship given by the following formula [4,5]:

$$\eta_t = A \cdot e^{-\frac{AE}{RT}}, \quad (1)$$

where A is dependent on the mass and molar volume, a constant characteristic for a given liquid, ΔE is the viscous flow activation energy, $R_C = 8.31446261815324 \text{ J}\cdot\text{mol}^{-1}\cdot\text{K}^{-1}$ is the Clapeyron gas constant, and T (K) is the absolute temperature ($T = t + 273.15$).

During engine operation, lubricating oil ages and becomes contaminated, leading to a change in its viscosity. The appropriate viscosity of the oil is obtained within the set temperature range at which the oil is operated. The temperature varies in different areas of the engine, resulting in a change in oil viscosity. It is, therefore, important that the change in viscosity be as small as possible [1]. This ensures that changes in oil temperature will induce correspondingly small changes in the rheological properties of the oil and ensures the smallest possible changes in the thickness of the lubricating film that the lubricating oil creates at the tribological nodes [6].

The standardized oil viscosity at reference temperatures makes it possible to classify oils by determining their viscosity at reference temperatures. For instance, the ISO Technical Committee (TC23) has defined 20 viscosity grades labeled as ISO VG [7], with the grade number representing a rounding to the nearest integer of the average kinematic viscosity expressed in units of mm^2/s (cSt) measured at $40 \text{ }^\circ\text{C}$, allowing for a viscosity tolerance of $\pm 10\%$ relative to the average.

Another commonly used viscosity classification is the J300 classification [8], which groups oils into summer oil and winter oil viscosity grades. Oils that simultaneously meet a certain summer and winter grade are multi-season oils. Assigning a particular oil to a specific summer grade is determined by the oil's kinematic viscosity (mm^2/s) at $100 \text{ }^\circ\text{C}$, which must fall within a specified range, and the minimum dynamic viscosity value is expressed in units of $\text{mPa}\cdot\text{s}$ at $150 \text{ }^\circ\text{C}$. Winter oils are classified based on minimum kinematic viscosity expressed in mm^2/s at $100 \text{ }^\circ\text{C}$ and two specific dynamic viscosities ($\text{mPa}\cdot\text{s}$); namely, the maximum low-temperature cranking viscosity at a standardized temperature and low-temperature pumping viscosity with no yield stress at a standardized temperature.

Since the viscosity of the oil changes with temperature, it is important that the oil used in the engine be of adequate viscosity grade but also exhibit the least possible viscosity changes due to changes in oil temperature [9]. A previous paper [10] presents the viscosity-temperature coefficient (VTC) developed based on ref. [11], which can be defined as the relative decrease in kinematic viscosity (mm^2/s) of the tested oil U and Y determined at $40 \text{ }^\circ\text{C}$ and $100 \text{ }^\circ\text{C}$, respectively, relating to the viscosity U as given by the relationship:

$$VTC = \frac{U - Y}{U}, \quad (2)$$

However, the mentioned index does not capture the exponential variation of viscosity as a function of temperature. The use of indicators determining the viscosity quotient at two different temperatures or the viscosity coefficient calculated as the difference in viscosity at two temperatures divided by the average value of these two viscosities is similarly limited [2]. Therefore, the variation of viscosity with temperature is commonly described by the viscosity index. This parameter is arbitrary and is based on the kinematic viscosity of the oil at 40 and $100 \text{ }^\circ\text{C}$, as well as the kinematic viscosity at $40 \text{ }^\circ\text{C}$ of two hypothetical oils that have a kinematic viscosity at $100 \text{ }^\circ\text{C}$ equal to that of the oil being tested at $100 \text{ }^\circ\text{C}$ (a hypothetical oil with $VI = 0$ and a hypothetical oil with $VI = 100$). Lubricating oils of the same viscosity grade may exhibit varying viscosity index values, as depicted in Figure 1. The smaller the oil's viscosity index, the greater the change in viscosity with temperature.

The standard procedure for determining the viscosity index of the tested oil in accordance with ASTM D2270 [12] and ISO 2909 [13] is presented in the form of an algorithm in . The procedure consists of the following steps:

1. Determination of the kinematic viscosity values at $40 \text{ }^\circ\text{C}$ of two hypothetical oils that have a kinematic viscosity at $100 \text{ }^\circ\text{C}$ equal to the kinematic viscosity of the oil under testing at $100 \text{ }^\circ\text{C}$. The kinematic viscosity at $40 \text{ }^\circ\text{C}$ is determined for oil with a conventionally assigned viscosity index of 0 ($VI = 0$), labeled L , and for oil with a viscosity index of 100 ($VI = 100$), labeled H . According to the aforementioned viscosity

standards, for a test oil with a kinematic viscosity at 100 °C labeled Y , which is greater than $2 \text{ mm}^2/\text{s}$ and smaller than $70 \text{ mm}^2/\text{s}$, the values of L and H are read from a table that is an integral part of the standard. If the oil under testing has Y greater than $70 \text{ mm}^2/\text{s}$, the values of L and H are calculated based on the following relationships:

$$L = 0.8353 \frac{\text{s}}{\text{mm}^2} \cdot Y^2 + 14.67 \cdot Y - 216 \frac{\text{mm}^2}{\text{s}}, \quad (3)$$

$$H = 0.1684 \frac{\text{s}}{\text{mm}^2} \cdot Y^2 + 11.85 \cdot Y - 97 \frac{\text{mm}^2}{\text{s}}, \quad (4)$$

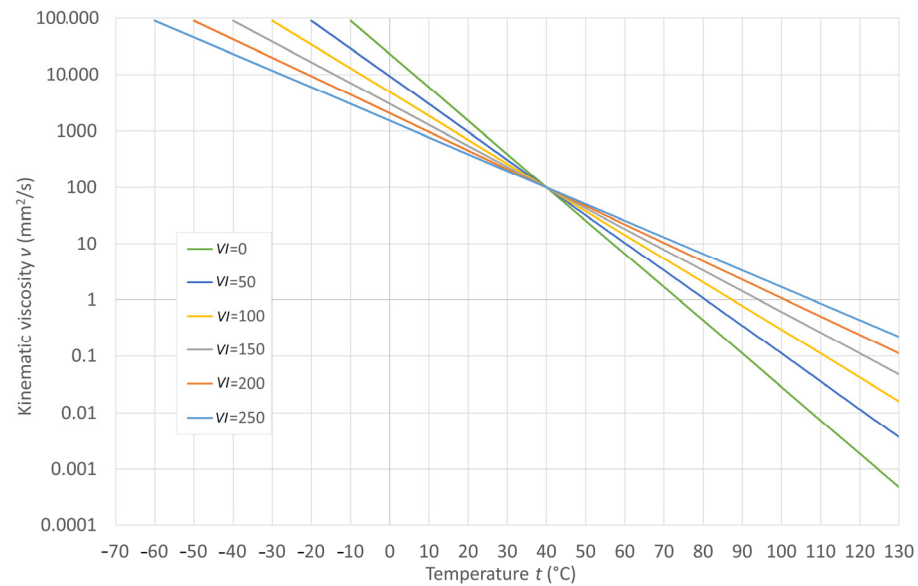


Figure 1. Example of viscosity variation with temperature of oils belonging to the same viscosity grade ISO VG 100, having different viscosity index values (based on ref. [9]).

- Determination of viscosity index values based on functions of the viscosity of the tested oil U at 40 °C for oil with a viscosity index less than or equal to 100, or functions of the viscosity of the tested oil U and Y at 40 °C and 100 °C, respectively, for oil with a viscosity index greater than 100. The viscosity index can be calculated using the following formula:

$$VI = \begin{cases} \frac{L-U}{L-H} \cdot 100, & \text{if } VI \leq 100 \\ 100 + \frac{\text{antilog}\left(\frac{\log H - \log U}{\log Y}\right) - 1}{0.00715}, & \text{if } VI > 100 \end{cases} \quad (5)$$

In most cases, the mentioned procedure requires the use of a table and interpolation for the read values of L and H , which results in an approximation error. Online calculators are available to determine the viscosity index. Among the most popular calculators that help calculate the viscosity index are the Anton Paar tools [14], ÖleZol [15], Evonik [16], Klüber Lubrication [17], or TriboNet [18]. However, these tools employ various interpolation methods and have different precisions in calculating relationship (9), leading to discrepancies in the results obtained using each of these tools. Furthermore, the calculators operate with various ranges of input data, and some restrict the input to only integers.

Given the limitations associated with determining the viscosity index and the challenge of describing the relationships provided in the tables of the mentioned standards, the authors have proposed a simplified method for roughly estimating the viscosity index. This method is based on a model that approximates the values L and H as functions $f(Y)$ and $g(Y)$ which depend on the viscosity of the tested oil at 100 °C.

The method presented in the article is based on the indicators proposed in ASTM D2270 and ISO 2909 standards and uses viscosity at 40 °C and 100 °C to determine the viscosity index. However, the proposed method does not require the use of reference oil tables indicated in the mentioned standards. The difference between these approaches is demonstrated in Appendix A. The proposed method shows a very small error, and, thus, it may be practical for use. It can be implemented in online lubricant oil quality assessment systems [19,20], for example, to monitor the dilution of lubricant oil with diesel oil during operation. One of the problems that may arise during the operation of internal combustion engines is the dilution of lubricant oil with diesel fuel. The authors have analyzed this issue in several previous publications [21,22]. Research has shown that the dilution of lubricant oil with fuel can not only accelerate destructive processes occurring in the engine but also increase the risk of explosions in the engine's crankcase [23,24]. This is due to the complex interaction associated with the formation of hot spots and the increased volatility of fuel-diluted lubricant oil [22]. One of the methods considered in previous studies to assess the degree of lubricant oil dilution with diesel fuel is the analysis of changes in lubricant oil viscosity during operation. However, viscosity depends not only on the chemical composition of the fluid but also on temperature. This article expands on these analyses and presents earlier results as an example of the application of the method proposed in this article.

2. Materials and Methods

Based on the presented assumptions and the recommendations outlined in the standards, the auxiliary functions required to determine the viscosity index are plotted, which are based on the following:

$$L = f(Y), \quad (6)$$

$$H = g(Y), \quad (7)$$

Graphs of the Functions (6) and (7) are depicted in Figure 2.

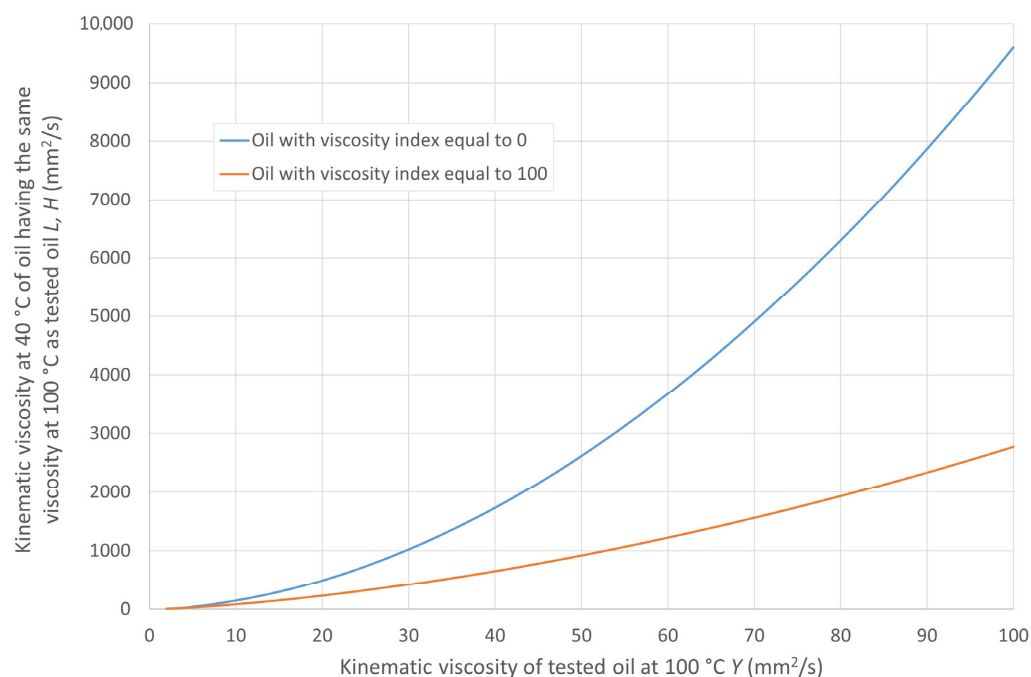


Figure 2. Relationships of the viscosity at 40 °C—of a hypothetical lubricating oil with VI = 0 and VI = 100 having the same viscosity at 100 °C as the presented test oil—as functions of the viscosity of the test oil at 100 °C.

To achieve a strong model fit, the authors suggest using 5th-degree polynomials in the following form:

$$L = a_1 \cdot Y^5 + a_2 \cdot Y^4 + a_3 \cdot Y^3 + a_4 \cdot Y^2 + a_5 \cdot Y + a_6, \quad (8)$$

$$H = b_1 \cdot Y^5 + b_2 \cdot Y^4 + b_3 \cdot Y^3 + b_4 \cdot Y^2 + b_5 \cdot Y + b_6, \quad (9)$$

where: $a_1 \left(\frac{s^4}{mm^8}\right)$, $a_2 \left(\frac{s^3}{mm^6}\right)$, $a_3 \left(\frac{s^2}{mm^4}\right)$, $a_4 \left(\frac{s}{mm^2}\right)$, $a_5 (-)$, $a_6 \left(\frac{mm^2}{s}\right)$, $b_1 \left(\frac{s^4}{mm^8}\right)$, $b_2 \left(\frac{s^3}{mm^6}\right)$, $b_3 \left(\frac{s^2}{mm^4}\right)$, $b_4 \left(\frac{s}{mm^2}\right)$, $b_5 (-)$, and $b_6 \left(\frac{mm^2}{s}\right)$ are the coefficients of approximating equations.

To reduce the fitting error of models (8) and (9), a spline function consisting of three input size ranges is utilized. These are given as follows:

1. The range of the lowest values of kinematic viscosity of the tested oil falling within $2.1 \text{ mm}^2/\text{s} \leq Y < 6.7 \text{ mm}^2/\text{s}$. The limit value of 6.7 is adopted for the accuracy of the model approximating the median range of kinematic viscosities indicated in the tables of the ASTM D2270 and ISO 2909 standards, below which the value of the relative percentage error of the proposed models is greater than 3%, which the authors adopted as the limit value.
2. The range of medians of kinematic viscosity of the tested oil is $6.7 \text{ mm}^2/\text{s} \geq Y \leq 70.0 \text{ mm}^2/\text{s}$.
3. The range of highest kinematic viscosity values of the tested oil $Y > 70.0 \text{ mm}^2/\text{s}$ is modeled by relations (3) and (4) (the range is derived from models indicated explicitly in ASTM D2270 and ISO 2909).

The analysis of the accuracy of the models' representation of empirical data was carried out in accordance with a previously developed approach. For each model, for all points in each of the analyzed ranges, the model determination coefficient R^2 (-) and the relative percentage error δ_i (%) is determined for each value $i = 1, 2, \dots, n$ in line with the following equations:

$$R^2 = \frac{\sum_{i=1}^n (\hat{x}_i - \bar{x})^2}{\sum_{i=1}^n (x_i - \bar{x})^2}, \quad (10)$$

$$\delta_i = \frac{|x_i - \hat{x}_i|}{x_i} \cdot 100\%, \quad (11)$$

where x_i is the i -th value of the analyzed parameter x (viscosity or viscosity index) adopted as an exact value, \hat{x}_i is the i -th value of x determined based on the model, and \bar{x} is the arithmetic mean of x values determined on the basis of the model.

The calculations were performed using Matlab R2024a software (MathWorks, Natick, MA, USA).

3. Results and Discussion

As a result of analyzing the values L or H indicated in the standards and for the assumptions made, the approximating coefficients for Equations (8) and (9) are determined, which provide the best fit in each of the analyzed ranges. The coefficients of the approximating equation are shown in Tables 1 and 2. An algorithm for calculating VI based on models (8) and (9) and the determined coefficients are shown in .

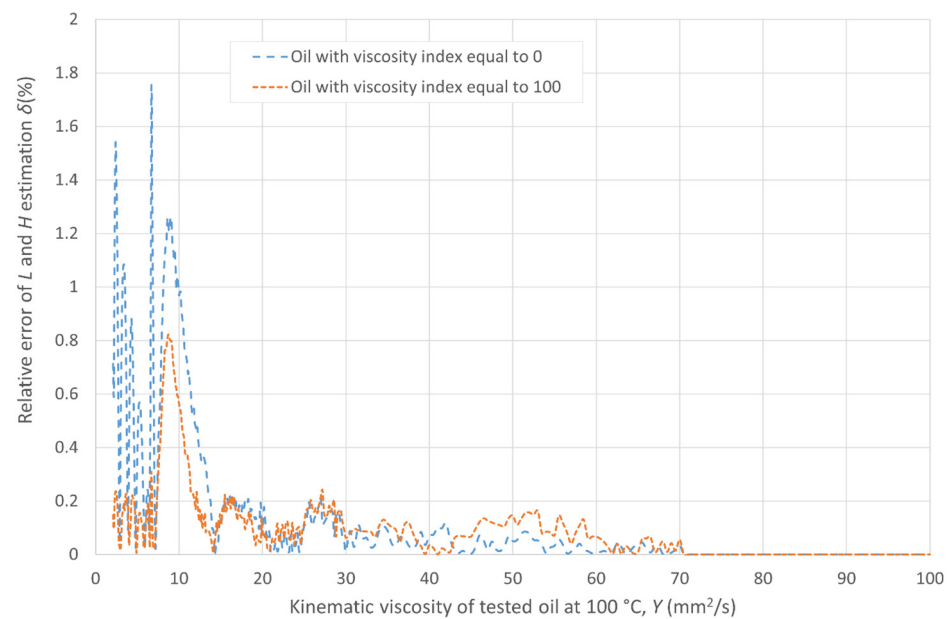
Table 1. Coefficients of the approximating equation L, the coefficient of determination, and the relative percentage error of the model (7).

Y	a_1	a_2	a_3	a_4	a_5	a_6
$\left(\frac{mm^2}{s}\right)$	$\left(\frac{s^4}{mm^8}\right)$	$\left(\frac{s^3}{mm^6}\right)$	$\left(\frac{s^2}{mm^4}\right)$	$\left(\frac{s}{mm^2}\right)$	(-)	$\left(\frac{mm^2}{s}\right)$
$6.7 > Y \geq 2.1$	0.0074	-0.3039	3.7190	-17.6970	43.6760	-33.9310
$70.0 \geq Y \geq 6.7$	-1.84×10^{-7}	3.20×10^{-5}	-0.0025	0.9966	6.1596	-12.7890
$Y > 70$	0	0	0	0.8353	14.6700	-216.0000

Table 2. Coefficients of the approximating equation H, the coefficient of determination, and the relative percentage error of the model (8).

Y	b_1	b_2	b_3	b_4	b_5	b_6
$\left(\frac{\text{mm}^2}{\text{s}}\right)$	$\left(\frac{\text{s}^4}{\text{mm}^8}\right)$	$\left(\frac{\text{s}^3}{\text{mm}^6}\right)$	$\left(\frac{\text{s}^2}{\text{mm}^4}\right)$	$\left(\frac{\text{s}}{\text{mm}^2}\right)$	(-)	$\left(\frac{\text{mm}^2}{\text{s}}\right)$
$2.1 \leq Y < 6.7$	0.0048	-0.1204	1.0693	-3.5619	10.0950	-6.3650
$6.7 \leq Y \leq 70$	-2.20×10^{-7}	5.03×10^{-5}	-0.0046	0.3921	5.5020	-7.7091
$Y > 70$	0	0	0	0.1684	11.8500	-97.0000

Figure 3 shows, for models (7) and (8) with the coefficients shown in Tables 1 and 2, the calculated relative error of model fitting compared to the values shown in ASTM D2270 and ISO 2909.

**Figure 3.** Relative percentage error of fitting the proposed models' L or H to the values indicated in ASTM D2270 and ISO 2909.

The maximum value of the fitting error is calculated for each of the analyzed ranges. The calculated coefficients of determination and maximum values of the relative percentage error of estimation for the proposed models are summarized in Table 3.

Table 3. Summary of indicators describing the fit of the proposed models L or H.

Range of Values		Model L		Model H	
Y	R^2	δ_{max}	R^2	δ_{max}	Y
(mm^2/s)	(-)	(%)	(-)	(%)	(mm^2/s)
$2.1 \leq Y < 6.7$	0.9999	<1.54	1.0000	<0.24	$2.1 \leq Y < 6.7$
$6.7 \leq Y \leq 70$	1.0000	<1.74	1.0000	<0.83	$6.7 \leq Y \leq 70$
$Y > 70$	1.0000	0.00	1.0000	0.00	$Y > 70$

The obtained values of the coefficient of determination indicate a very good to complete fit of the proposed models, while the values of the relative percentage model fitting error over almost the entire range of kinematic viscosity of oil ($\geq 2 \text{ mm}^2/\text{s}$) are less than 2%, which is sufficient for the purpose of a preliminary or rough calculation of the viscosity index.

4. Example Application of the Model

To verify the suitability of the proposed approach for determining the viscosity index of lubricating oils of known kinematic viscosity at 40 and 100 °C, VI values calculated using the proposed approach and values calculated manually according to ASTM D2270 and ISO 2909 are compared. This is undertaken for an example of two datasets derived from viscosity measurements of mixtures of two single-season lubricating oils of viscosity grades SAE 30 and SAE 40 with diesel oil at concentrations of diesel oil in mixtures equal to 0, 1, 2, 5, 10, 20, and 50% *w/w*. As the viscosity values of the various mixtures change, the example presented is representative due to the possibility of assigning the mixtures tested to different viscosity grades, as shown in Table 4. The kinematic viscosity ranges at 40 °C of petroleum products belonging to the various ISO VG grades are presented in the literature on the subject [25].

Table 4. ISO viscosity grades of the mixtures used to verify the analytical method for determining the viscosity index.

SAE Viscosity Grade of Lubricating Oil Used to Prepare the Mixture [8]	Diesel Oil Content in the Lubricating Oil Mixture	Viscosity of the Mixture at 40 °C	ISO Viscosity Grade of the Mixture or the Limit Grades between Which the Mixture Is Rated [7]
SAE Grade	C	<i>U</i>	ISO VG
(–)	(% <i>w/w</i>)	(mm ² /s)	(–)
SAE 30	0	105.01	ISO VG 100
	1	98.06	ISO VG 100
	2	96.64	ISO VG 100
	5	82.70	[ISO VG 68, ISO VG 100]
	10	59.61	[ISO VG 46, ISO VG 68]
	20	36.25	[ISO VG 32, ISO VG 46].
	50	11.07	[ISO VG 10, ISO VG 15].
	100	2.897	ISO VG 3
SAE 40	0	159.90	ISO VG 150
	1	157.95	ISO VG 150
	2	149.10	[ISO VG 100, ISO VG 150]
	5	123.95	[ISO VG 100, ISO VG 150]
	10	78.71	[ISO VG 68, ISO VG 100]
	20	50.29	ISO VG 46
	50	13.25	[ISO VG 10, ISO VG 15].
	100	2.897	ISO VG 3

The results obtained using this analytical approach are compared with those calculated manually according to the mentioned standards [12] and those calculated using the Anton Paar calculator [14]. Measurement data for validation are taken from a previous study [22] which used Emi/Agip Cladium 120 SAE 30 CD/CF [26] and Emi/Agip Cladium 120 SAE 40 CD/CF [27] lubricating oils and Orlen Efecta Diesel Bio (designation CN27102011D) [28] diesel oil complying with ZN-ORLEN-5 [29]. The variation in viscosity for the mixtures tested as a function of diesel oil concentration and temperature is shown for SAE 30 and SAE 40 grade oil mixtures in Figures 4 and 5, respectively. The variation of viscosity index values depending on the composition of the mixture is shown in Figure 6.

Tables 5 and 6 show a comparison of the viscosity index values calculated using three different methods for mixtures of SAE 30 and SAE 40 viscosity-grade lubricating oil with diesel oil. The viscosity index is calculated manually using the method described in the standards, the Anton Paar calculator, and the algorithm proposed in this article. The percentage relative error of estimation using the proposed model compared to the other two methods is also presented, assuming that the other mentioned methods are treated as VI exact values.

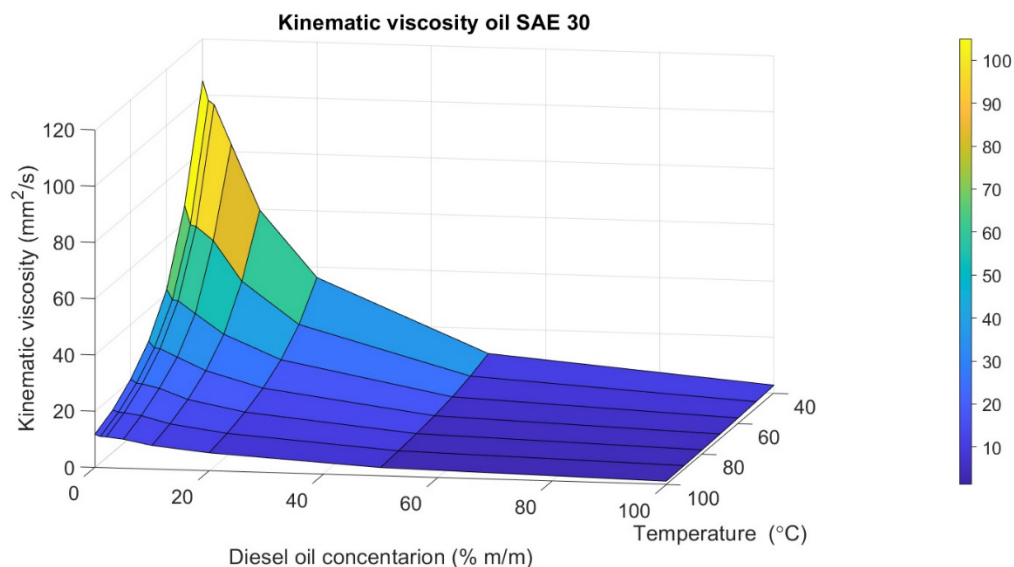


Figure 4. Dependence of kinematic viscosity of the tested mixtures of lubricating oil of SAE 30 grade with diesel oil on the composition and temperature of the mixture.

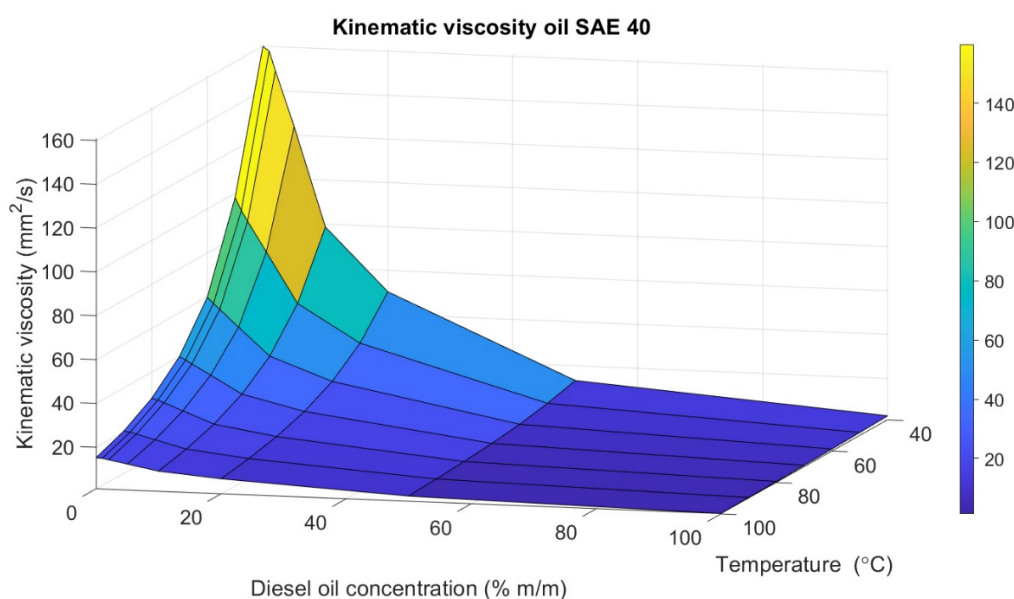


Figure 5. Dependence of kinematic viscosity of the tested mixtures of lubricating oil of SAE 40 grade with diesel oil on the composition and temperature of the mixture.

For mixtures of SAE 30 grade lubricating oil with diesel fuel, the maximum error in the tested measurement range is 1.02% when comparing the results obtained from the proposed analytical model to those attained using the Anton Paar calculator. In contrast, comparing the results of the analytical model application with a manual calculation based on standards yields a maximum percentage error of fitting of 1.12%. In turn, the coefficient of determination is determined for the sets of viscosity index values obtained by each method. When comparing the results from the model application to those from the Anton Paar calculator, the coefficient of determination R^2 is 0.9995. Similarly, when comparing the results from the model application to those obtained by manual calculations according to ASTM D2270/ISO 2909, the coefficient of determination is $R^2 = 0.9993$.

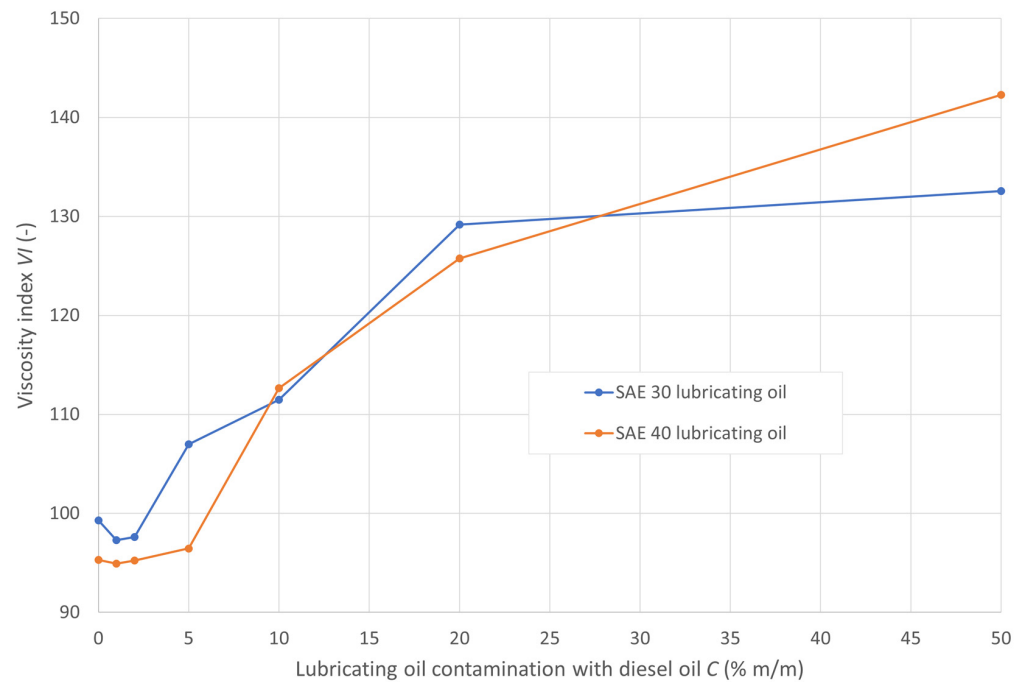


Figure 6. Viscosity index values of the tested lubricating oil–diesel mixtures from the composition, calculated in accordance with ASTM D2270/ISO 2909.

Table 5. Viscosity index values of the tested mixtures of SAE 30 grade lubricating oil with diesel oil calculated by different methods, and the percentage relative error of model fitting to other methods.

SAE 30 Grade Oil Based Mixtures		Viscosity Index VI Determined by a Method			Percentage Relative Error between Methods	
With diesel oil concentration of C	Manual calculation according to ASTM D2270/ISO 2909	Calculation with Anton Paar calculator	Calculation using an analytical model	Analytical model vs. Anton Paar calculator	Analytical model vs. manual calculation according to ASTM D2270/ISO 2909	
(% w/w)	(–)	(–)	(–)	(%)	(%)	
0	99.27	99.28	98.90	0.38	0.38	
1	97.18	97.29	96.46	0.85	0.74	
2	97.49	97.62	96.78	0.86	0.73	
5	106.76	106.97	106.04	0.88	0.67	
10	111.58	111.46	110.33	1.02	1.12	
20	128.91	129.17	129.20	0.03	0.22	
50	132.37	132.57	132.25	0.24	0.09	

Table 6. Viscosity index values of the tested mixtures of SAE 40 grade lubricating oil with diesel oil calculated by different methods, and the percentage relative error of model fitting to other methods.

SAE 40 Grade Oil Based Mixtures		Viscosity Index VI Determined by a Method			Percentage Relative Error between Methods	
With diesel oil concentration of C	Manual calculation according to ASTM D2270/ISO 2909	Calculation with Anton Paar calculator	Calculation using an analytical model	Analytical model vs. Anton Paar calculator	Analytical model vs. manual calculation according to ASTM D2270/ISO 2909	
(% w/w)	(–)	(–)	(–)	(%)	(%)	
0	95.25	95.44	94.89	0.58	0.38	
1	94.83	95.05	94.48	0.60	0.37	
2	95.21	95.35	94.83	0.54	0.41	
5	96.46	96.30	95.94	0.38	0.54	
10	112.44	112.65	111.65	0.89	0.70	
20	125.78	125.78	124.82	0.77	0.77	
50	142.33	142.33	141.67	0.46	0.46	

For mixtures of SAE 40 grade lubricating oil with diesel oil, fitting the results of the analytical model application to the other two methods yields maximum relative error results of 0.89% and 0.77%, respectively. In turn, the coefficient of determination is determined for the sets of viscosity index values obtained by each method. When comparing the results from the model application to those from the Anton Paar calculator, the coefficient of determination R^2 is 0.9999. Similarly, when comparing the results from the model application to those obtained by manual calculations according to ASTM D2270/ISO 2909, the coefficient of determination is $R^2 = 0.9999$.

5. Conclusions

The application of the method presented in this article showed that the proposed approach for the empirical data set analyzed provides a very good fitting between the results of the calculated VI value and those obtained by manual calculation based on the tables contained in ASTM D2270/ISO 2909 and by using the Anton Paar calculator.

The analytical method for calculating the viscosity index can be applied to the study of mixtures of lubricating products and fuels, particularly in examining the properties of their mixtures. For example, this method can be used to analyze lubricating oils enriched with suitable additives, lubricating oils diluted with fuel, and mixtures of various types of fuels.

The calculators of viscosity index use the tables provided in ASTM D2270 and ISO 2909 standards. While this approach is sufficient for calculating a single viscosity index value, algorithmic methods, including the one proposed in this article, may be more useful for applications in continuous oil quality monitoring systems.

The proposed method can be further refined in future work by increasing the number of ranges for which separate values of polynomial coefficients are determined to calculate L and H , which are intermediate indicators used to calculate viscosity index values.

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List of Abbreviations and Symbols

A	empirical parameters for determining dynamic viscosity
$a_1, a_2, a_3, a_4, a_5, a_6$	coefficients of equations approximating the viscosity L
ASTM	standard of ASTM International, formerly known as the American Society for Testing and Materials
$b_1, b_2, b_3, b_4, b_5, b_6$	coefficients of equations approximating the viscosity H
ΔE	activation energy of viscous flow
e	Euler number

$f(Y)$	empirical function to determine the viscosity L
$f(Y)$	empirical function to determine the viscosity H
H	kinematic viscosity at 40 °C of a hypothetical oil with $VI = 100$, which has (at 100 °C) a kinematic viscosity equal to that of the oil under test at 100 °C
ISO	International Organization for Standardization
L	kinematic viscosity at 40 °C of a hypothetical oil with $VI = 0$, which has (at 100 °C) a kinematic viscosity equal to that of the oil under test at 100 °C
R_C	Clapeyron's gas constant
R^2	coefficient of determination
SAE	SAE International, formerly the Society of Automotive Engineers
SAE 30, SAE 40	viscosity grades of lubricating oil according to SAE J300 standard
T	absolute temperature in K
t	relative temperature in °C
TC23	Technical Committee 23 of ISO
U	kinematic viscosity of the tested oil at 40 °C
VG	ISO viscosity grade
VI	viscosity index
VTC	viscosity–temperature coefficient
x	analyzed parameter (viscosity or viscosity index)
\bar{x}	arithmetic mean of x determined from the model
x_i	the i -th value of the analyzed parameter x
\hat{x}_i	the i -th value of x determined from the model
Y	kinematic viscosity of the tested oil at 100 °C
$\dot{\gamma}$	shear rate
δ_i	relative percentage error
η_t	dynamic viscosity at t
ρ_{15}	density at 15 °C.
ρ_t	density at t
τ	shear stresses

Appendix A

Algorithms

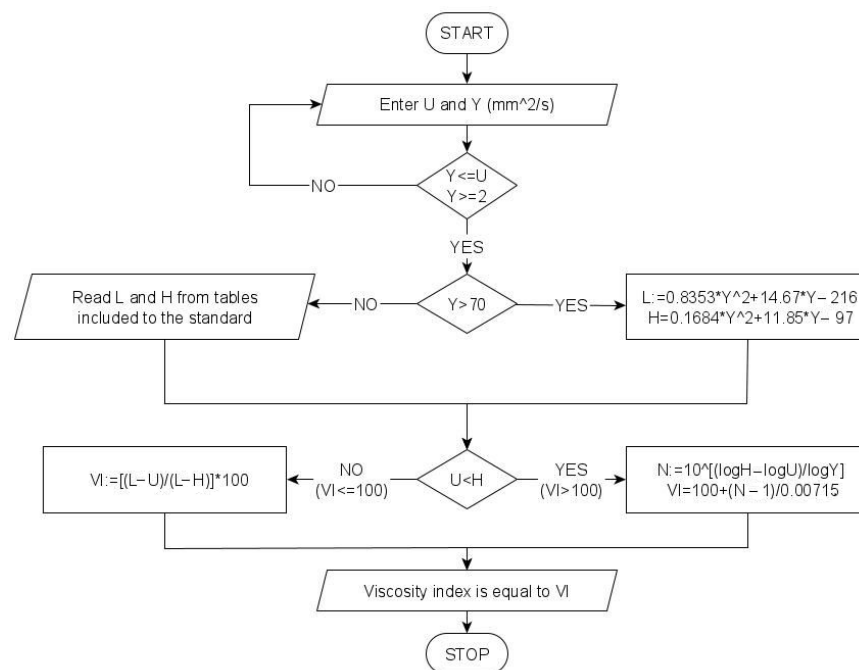


Figure A1. Standard procedure for determining viscosity index according to ASTM D2270 and ISO 2909.

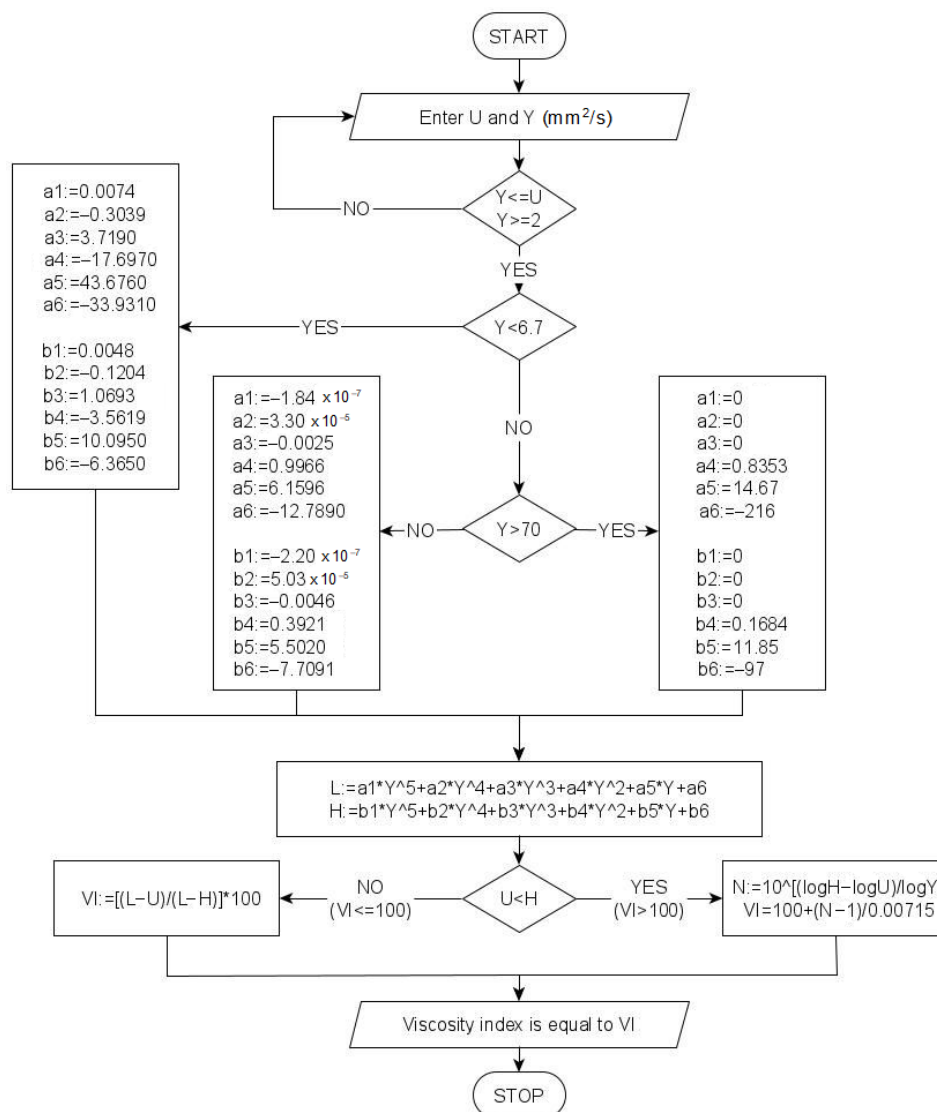


Figure A2. Developed analytical procedure for determining viscosity index.

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