Leszek CHYBOWSKI, Zbigniew MATUSZAK
Maritime Academy, Szczecin

ON CALCULATING ALGORITHMS IN RELIABILITY ANALYSIS

Key words

Reliability systems, factoring algorithms, inclusion-exclusion algorithms, sum of disjoint products algorithms.

Summary

In the presented paper, calculating algorithms applied to technical systems reliability analyses have been discussed. Algorithms' division and their characteristics have been presented. The basic source literature for the discussed algorithms has also been quoted.

Introduction

For the complete evaluation of a technical system operational reliability, it is necessary to estimate the values of the selected reliability measures. However, before the proper calculation stage, it is indispensable to carry out several activities as follows: Making the assumptions, system boundaries definition and its elements, defining and describing reliability structure system, choosing the mathematical model and estimating reliability measures describing system elements.

An important term describing each system – meant as a set comprising all elements interconnected by means of their relations and creating a system of two-direction interaction with the environment – is its state. The state of the technical system is defined as a set of all parameter values determining the system at a certain moment *t*. The set of possible technical systems' states is very numerous, theoretically its power is continuum. Therefore, in practical application, all possible object states are usually divided into a certain limited number of classes. The basis for the division classification is usage demands, not taking into account differences between various states within the scope of the same state class.

In order to avoid complications with reliability models, the technical objects' states are often divided only into two groups (binary systems); that is, success (up) and fault (down) states. In spite of the fact that technical systems and their elements may be in indirect up states, so called *partial up states,* which in many cases appear to be very significant for the correct real object activity description, they have not been directly dealt with in the paper, because they do not apply to the presented algorithms. Indirectly, the mentioned states may be described by means of the models presented in the paper after additional elementary events related to the system states have been introduced to the binary system reliability structure description.

1. Algorithms in reliability analysis

Binary technical system and its elements may be in one of two states: the direct application of the events' description system work analysis with the use of Boolean logic being possible. For the discussed Boolean model of system reliability structure, the unavailability function calculations can be done by means of applying particular presented algorithms. The term *algorithm* needs to be explained. It is often related to the name of the ancient Greek mathematician *Euclides* who lived 365-300 B.C.. He presented the method of calculating the biggest common divider for two integers *a* and *b (BCD).* However, the most probable word source for that term is the name of a 9th century Persian writer and mathematician *Abu Ja'far Mahammed ibn Musa al-Khowarizmi*, whose last part in Latin read as *Algorismus* [1]. The mathematician presented several rules explaining step by step arithmetical operation principles referring to digital numbers. At present the word algorithm denotes the following:

– a limited sequence of rules which apply to a limited number of data allowing to solve similar problems, and
– a set of rules characteristic for certain calculations or information technology activities.

The term algorithm here denotes a precisely defined course of activity aiming at evaluating system quantitative or qualitative measure within limited time based on the data connected with the system elements, that is, input data which belong to exactly defined set.

The starting point of all algorithms for calculating reliability measures is an initial analysis of the system reliability structure in order to transform it into a form required for carrying out calculations. The system structure is usually presented in a graphic form; e.g. in the form of a graph, fault tree, reliability block diagram, etc.

Most of the algorithms require the knowledge of all (for precise calculations) or a part of minimal system path sets or cut sets (for approximate calculations). The term *cut set* refers to a set of primary events (dynamic changes in the system elements structure and functioning which do not undergo any further decomposition at this stage of the analysis), whose simultaneous appearance causes system down. The cut set is called minimal if the set of its events can not be reduced without losing the cut set status. System *path set* is a set of primary events whose simultaneous appearing provides the system up state. The path set is called minimal if the set of its events can not be reduced without losing the status of path set.

In case of many analytical methods, the first stage is determining the system minimum cut sets (path sets) by means of a specified algorithm for searching for system minimum paths or cut sets which is referred to as *qualitative analysis algorithm.* After that, it is possible to carry out the proper calculations using the calculating algorithm referred to as *quantitative analysis algorithm*.

Determining the system minimum paths in a direct way is based on relatively simple algorithms depending upon the available system structure description with different searching method graphs like algorithms by *Deo, Lipski, Kulikowski* or making use of Boolean matrix operations like the *Rajnske* algorithm. The technical system minimal path sets searching methods have been precisely described in literature [2–4].

As far as determining the minimal cut sets is concerned, historically, the first algorithm widely described and still used has been worked out by *Fussell and Vesely* in the beginning of 1970s and named *MOCUS (Method of Obtaining Cut Sets).* The algorithm is based on the fault tree transformations of the system using the *Top-Down* evaluation method.

The fault tree may also be analysed by means of the *Bottom-Up* evaluation method. The authors of the first algorithm of that type, called *MISCUP (Minimal Cut Set Upward*), were *Pande, Spector and Chattarjee.* There have been various methods of determining minimal cut sets. One of the first ones was *FATRAM,* a more effective algorithm than *MOCUS,* worked out by *Rasmuson and Mashall.* In minimal cut set searching process direct, usually complex searching methods like algorithms by *Abel, Ahmad, Allan, Biegel, Pearson* are applied [5-7]; or, on the basis of minimal paths knowledge. *Inverting algorithms* connected with binary modelling property, which makes it possible to determine all minimal cut sets when all system minimal path sets have been known and vice versa (duality principle).

So far many calculating algorithms based on minimal path sets or cut sets have been created. In fact, minimal path sets algorithms are equal to those based on minimal cut sets knowledge because, as it has been mentioned, when defining inverting algorithms according to the duality principle, transforming of one algorithm into another is possible.

It is worth emphasising that there have existed methods of technical system reliability analysis excluding system minimal cut sets or paths searching process, and the methods have been presented in [8, 9].

However, it can be observed that the majority of algorithms are based on minimal cut sets or path sets of the analysed system. Among many calculations of algorithms, two main groups, that is, simulating and analytical algorithms, need to be distinguished.

Simulating algorithms are based on stochastic simulation (Monte Carlo method). The algorithms belong to mathematical problem solving methods making use of stochastic modelling, which deals with matching a problem to be solved with a random process whose statistic parameters make the searched values of the problem approximate. So far, many algorithms of that kind have been created. Among the most important ones, algorithms by *Easton, Fishman, Kamat, Kumamoto, Henley, Kalyan, Kubat* can be pointed out [10].

Analytical algorithms make use of relations between particular reliability measures; whereas, the relations, as such, are possible to be expressed by means of mathematical formulas. They result in unambiguous, although not always very precise, solutions. Solving problems by means of analytical methods takes less time and is less complex than by means of simulating algorithms.

Conclusions

According to the following calculation methodology, the criteria of analytical algorithms may be divided into several main groups:

– IE-algorithms *(Inclusion-Exclusion algorithms)* based on minimal system path sets or cut sets and making use of Poincare formula. They are the most often applied for calculating reliability measures, mainly because they are relatively simple. There have existed many inclusion-exclusion algorithms, like algorithms by *Caverse*, *Dhillon*, *Nelson*, *Locks*, *Shooman* [10]. Their disadvantage is the rapid increase of calculation time with the number of the system minimal cut sets or path sets**.** In such situations, it is possible to make use of inclusion-exclusion algorithms, focusing on Bonfferoni inequalities and their improvements. Algorithms by *Fong* and *Buzacott*, *Dziubdziela* and *Kopciński*, *Heidtmann, Hunter*, *Worsley* [11–13] belong to the best known algorithms of that type.
– SDP-algorithms (*Sum of Disjoint Products algorithms*) are based on generating the system structure as a sum of disjoint products. The algorithms generate pairs of Boolean variable disjoint products, which together give the function of the system structure. Many algorithms of that kind have been created, and some of them have been presented in [14, 15]. Most *SDP-*

algorithms require the knowledge of all minimal path sets or cut sets. One of the most effective is the *Abraham* algorithm [14], which generates consecutive Boolean variable products in such a way that each newly generated product is disjoint in relation to the previous ones. The algorithm has also been improved by means of reducing the number of generated disjoint products and by the application of inverting algorithms for generating minimal paths and cut sets of the system. Another commonly known is the *Dotson* and *Gabien* algorithm [10], which determines system minimal paths as far as it is needed and does not require their knowledge before the calculation process is started. Moreover, the algorithm generates a *SDP* form of the system structure function simultaneously with its supplement, which allows achieving two-sided system availability assessments without the need for generating all products. The *Dotson-Gabien* algorithm is applied to graph modelled systems.

– Factoring algorithms, called linear decomposition algorithms, are based on Shannon decomposition referred to a selected element. They work on the basis of linear system decomposition in relation to consecutive elements as long as it is possible to achieve a trivial system or a system for which reliability indicators may be easily assessed, for example, series-parallel, series, one-element system etc. Mostly, the algorithms do not require the knowledge of system minimal cut sets or path sets, among others, except the cases when minimal path sets or cut sets are used as a tool for determining the system state after a consecutive factoring or a criterion for factoring process completion. The system description is usually done in a graphic form, e.g. a block, a graph or a disability tree. So far, a considerable amount of published material has been devoted to factoring algorithms mainly for systems modelled by means of a graph like [16, 17]. Factoring algorithms in connection with an adequate system structure reduction are considered to belong to the most effective ones. The *ERAC (Exact Reliability/ Availability)* calculation algorithm by *Aven* is a popular one in the group. It is based on the decomposition method by *Doulliez* and *Jamoulle* designed for transportation networks. The *Aven* algorithm has also got modified versions.
– Other algorithms are based on other methods different from the above. Thus, we may distinguish algorithms based on minimal path sets or cut sets which deal with Boolean matrix operations making use of various logical operations in order to achieve a system structural function convenient for making substitutions and various types of mixed algorithms, e.g., the algorithm by *Hariri* and *Raghavendra*, which makes use of the *SDP*-algorithm and conditional probability. Moreover, there are a number of algorithms that do not require system minimal paths or cut sets knowledge. These algorithms are based on system graphic description with the use of a graph or a fault tree model; thus, we may distinguish so called signal methods with a possible graph transformation and the method of a graph transformed into a fault tree.

Recently, dynamic fault trees have been developed. The trees may be based on special additional dynamic gates or make use of dynamic event description, not as in a static fault tree analysis the model of event occurrence constant probability. A more detailed description of the topic may be found in [10, 18]. Other graph methods making use of graph transformations and decomposition have been extensively presented in [19, 20]. A separate group comprises algorithms dealing with Markov chain processes [21, 22]. Among many algorithms, the ones that make use of reliability measures' assessments need to be given special attention. The assessments, as already mentioned, allow for IE-algorithms' improvement (due to *Bonferronni* inequality). Reliability measure assessments may be used to build algorithms (not necessarily based on inclusion-exclusion method). Series and parallel assessment, *Esary-Proschan*, *mini-max* and *Litwak-Uszakov* assessments, belong to the best known ones [10, 23–25]. Both *Esary-Proschan* and series-parallel assessments allow achieving upper bound approximation of unavailability formula.

References

1. Wróblewski P.: Algorithms, data formats and software engineerings. Helion, 2001 (in Polish).
2. Deo N.: Graph theory and its application in technology and computer science. PWN, Warszawa, 1980 (in Polish).
3. Kubat P.: Estimation of reliability for communication/computer networks – simulation/analytic approach. IEEE Transactions on Communications, 1989, 9, 927–933.
4. Lee S.H.: Reliability evaluation of a flow network. IEEE Transactions on Reliability, 1980, R-29, 24–26.
5. Apostolakis G.: How useful is quantitative risk assessment? Risk analysis 2004, 24(3).
6. Ahmad S.H., Jamil A.T.M.: A modified technique for computing network reliability. IEEE Transactions on Reliability, 1987, 5, 554–556.
7. Biegel J.E.: Determination of tie-sets and cut-setsfor a system without feedback. IEEE Transactions on Reliability, 1977, 1, 39–42.
8. Brown K.S.: Evaluating fault trees (AND & OR gate only) with repeated events. IEEE Transactions on Reliability, 1990, 2, 226–235.
9. Page L.B., Perry J.E.: A simple approach to fault-tree probabilities. Computers and Chemical Engineering, 1986, 10, 249–257.
10. Karpiński J., Korczak E.: Methods of estimation of technical objects reliability. Omnitech Press, PAN, 1990 (in Polish).
11. Diubdziela W., Kopociński B.: Estimation of the systems' reliability. Matematyka Stosowana, 15, 61–73 (in Polish).

12. Fong C.C., Buzacott J.A.: Improved bounds for system-failure probability. IEEE Transactions on Reliability, 1987, 4, 454–458.
13. Heidtmann K. D.: Bounds on reliability of a noncoherent system using its length & width. IEEE Transactions on Reliability, 1982, 5, 424–427.
14. Abraham J.A.: An improved method for network probability. IEEE Transactions on Reliability, 1979, 1, 58–61.
15. Aggarwal K.K., Misra K.B., Gupta J.S.: A fast algorithm for reliability evaluation. IEEE Transactions on Reliability, 1975, 1, 83–85.
16. Agraval J.A., Barlow R.E.: A survey on network reliability and domination theory. Operations Research, 1984, 32, 478–492.
17. Dotson W.P., Gobien J.O.: A new analysis technique for probabilistic graphs. IEEE Transactions on Circuits and Systems, 1979, 10, 855–865.
18. Abel U., Bicker R.: Determination of all minimal cut-sets between a vertex pair in an undirected graph. IEEE Transactions on Reliability, 1982, 2, 167–171.
19. Abo El Fotoh H.M., Colbourn C.J.: Computing 2-terminal reliability for radio-broadcast networks. IEEE Transactions on Reliability, 1989, 5, 538–555.
20. Beichelt F.: Zurverlässigkeit Strukturierer Systeme. VEB Verlag Technik, Berlin, 1988.
21. Bailey M.P., Kulkarni V.G.: A recursive algorithm for computing exact reliability measures. IEEE Transactions on Reliability, 1986, 1, 40–56.
22. Girtler J.: Reliability of the engine in case of damages eliminate. Zagadnienia Eksploatacji Maszyn, 1992, 1, 75–80 (in Polish).
23. Rausand M., Høyland A.: System reliability theory: models, statistical methods, and applications. Second edition. New Jersey, Wiley, Interscience, 2004.
24. Saleh J. H., Marais K.: Reliability: How much is it worth? Beyond its estimation or prediction, the (net) present value of reliability. Reliability Engineering and System Safety, 2006, 91, 665–673.
25. Wang H., Sevcik K. C., Serazzic G., Wang S.: The general form linearizer algorithms: A new family of approximate mean value analysis algorithms. Performance Evaluation, 2008, 65, 129–151.

Recenzent:
Zenon ZWIERZEWICZ

O algorytmach obliczeniowych w analizie niezawodnościowej

Słowa kluczowe

Niezawodność systemów, algorytmy faktoryzacji, algorytmy włączeń i wyłączeń, algorytmy sum rozłącznych iloczynów.

Streszczenie

W materiale dokonano przeglądu algorytmów obliczeniowych stosowanych w analizach niezawodnościowych dla systemów technicznych. Przedstawiono podział algorytmów oraz zaprezentowano ich charakterystyczne cechy. Przytoczono podstawową literaturę źródłową dla wymienionych algorytmów.