RELIABILITY IMPORTANCE ANALYSIS OF TECHNICAL SYSTEMS ELEMENTS

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Selected qualitative and quantitative measures for reliability importance evaluation have been presented. Measures, which can be applicable for elements and groups of elements in technical systems, have been shown.

1. Introduction

Very often as one of steps in reliability analysis, it is necessary to determine which elements or cut sets are the most important for system, on account of optimal value of selected dependability measure assurance [6].

These issues are connected to problem of searching for *weak links* in the system, and it is called *importance analysis*. From dependability point of view, importance of given element in the system is depend on two factors:

1. Reliability characteristics of the element.
2. Reliability structure in which the element is located.

Influence of first factor is obvious. In relation to element location in reliability structure, the element is the more important, the element is more similar to single item inserted in serial reliability structure of system. Influence of element on system reliability is decreasing with element redundancy level increasing.

In qualitative analyses, importance of minimal cut set usually depends on number of elements in this set. This number is called *order of minimal cut set*. Very often cut set of first order is more important (critical) then cut sets of higher orders. If system has cut set with one element only, then fault of this element is bringing on down state of the system. This case is related to elements in serial reliability structures.

Order of smallest cut set with i-th elements is given by qualitative measure $I^O(i)$. Let C_{1i}, C_{2i}, ..., C_{ni} are describing all cut sets with event E_i, then:

$$I^O(i) = \min_{k=1i,2i,...ni} [card(C_k)] \qquad (1)$$

Value of $I^O(i)$ does not depend on the component reliabilities. For analysis systems modelled by means of fault tree, can be useful similar coefficient with is giving numbers of occurrences i-th events in the fault tree [4]. Usually element is the more important, the element exist in more number of cut sets.

Other important factor in qualitative analysis of element important is ranking of primary events in given cut set [11]. For instance, it can be depend on assumption that, human faults are more frequent then failures of active elements, and failures of active elements are more frequent then failures of passive elements. Based on ranks of elements, it is possible to build rankings of two or more events minimal cut sets consisted of different kind of events [3]. Qualitative methods are useful for systems modelled with binary function of system structure [5].

Matuszak and Kołodziejski proposed other qualitative measure [8]. According to them importance of element is characterised by sum of energetic fluids streams $I^{KM}(i)$ (streams measure) which are on input s_i and on output s_o from i-th element of technical system. It can be presented by formula:

$$I^{KM}(i) = s_i(i) + s_o(i) \tag{2}$$

Measure this can be represented by value from range <0,1>:

$$I^M(i) = k_{KM} I^{KM}(i) = k_{KM}[s_i(i) + s_o(i)] \tag{3}$$

Where: $k_{KM} = [\sum_{i=1}^{n} I^{KM}(i)]^{-1}$ - coefficient which is providing summing to the one. n – number of elements in the system.

For quantitative analysis of importance, there are introduced *measures of* importance. It is number of these measures, which application is depend on importance aspect, which is developed. Different measures have different definitions, so these are providing different importance rankings.

Usually it is necessary to find elements (importance measures of elements) which dependability measures should be improved for increase reliability increase of whole system. Analogically it is possible to analyse importance of cut sets (locally importance measures).

Qualitative ranking of minimal cut sets is based on measure called cut set importance. Unavailability of cut sets quantifies the probability that k-th cut set is in failed state at a time t:

$$Q_k(t) = q_{1,k}(t) \cdot q_{2,k}(t) \cdot \ldots \cdot q_{j,k}(t) = \prod_{i=1}^{j} q_{i,k}(t) \tag{4}$$

The cut set importance can be interpreted as the conditional probability that minimal k-th cut set is failed at time t, given that the system is failed at time t. The cut set importance is calculated as:

$$I^{CI}(k,t) = \frac{\breve{Q}_k(t)}{Q_0(t)} \tag{5}$$

Where: $Q_0(t)$ – unavailability of system.

2. Selected quantitative importance measures

Some of importance measures for elements has been presented below. Measures can be applicable for repairable and non-repairable systems. Authors selected measures: Birnbaum's measure of reliability importance, Vesely-Fussell's measure of reliability importance, improvement potential, Lambert's criticality importance, Birnbaum's measure of structural importance.

Historically first measure has been proposed by Birnbaum [2]. Let $\bar{r}(t) = [r_1(t), r_2(t), \ldots, r_n(t)]$ is system elements reliability vector in moment t, and $R[\bar{r}(t)]$ is system reliability, which is depend on reliability of all elements and reliability structure of system. Birnbaum's measure for i-th element is given as:

$$I^B(i \mid t) = \frac{\partial R[\bar{r}(t)]}{\partial r_i(t)} = \frac{\partial F[\bar{r}(t)]}{\partial f_i(t)} \tag{6}$$

Where: $F[\bar{r}(t)] = 1 - R[\bar{r}(t)]$ is unreliability function of system in moment t, and $f_i(t)$ is probability density function of time to i-th element.

Measure of Birnbaum for i-th element in moment t, can be represented analogically by unavailability functions:

$$I^B(i \mid t) = \frac{\partial Q_0(t)}{\partial q_i(t)} = \frac{\partial Q[\bar{q}(t)]}{\partial q_i(t)} \tag{7}$$

Where: $\overline{q}(t)=[q_1(t),q_2(t),...,q_n(t)]$ – vector of unavailability of system elements in moment t, $Q_0(t)=Q[\overline{q}(t)]$ – unavailability of system.

For preliminary analysis can be use formula given in [1]:

$$I^B(i\mid t)\approx\frac{\sum_{j=1}^{m_i}\breve{Q}_j(t)}{q_i(t)} \tag{8}$$

Where: $\breve{Q}_j(t)$ – unavailability of j-th cut set, which contains of i-th element, $m_i(t)$ – number of cut sets, which consist of i-th element, $q_i(t)$ – unavailability of i-th element.

Birnbaum's measure can be calculated as the difference between the probabilities of system failure event calculated under the assumptions that i-th element is known to occur and is known to not occur, respectively. This difference may be interpreted as the probability that input event no. i is critical at time t:

$$I^B(i\mid t)=\frac{\partial Q_0(t)}{\partial q_i(t)}=Q[q_i(t)=1,\overline{q}(t)]-Q[q_i(t)=0,\overline{q}(t)] \tag{9}$$

Vesely-Fussell's measure of reliability importance $I^{VF}(i|t)$ for component i is defined as the conditional probability that at least one minimal cut set containing i-th element is failed at time t, given that the system fails at time t.

Let m_i is describing number of minimal cut sets with i-th element; $C_{ij}(t)$ – j-th minimal cut set, which consist of i-th element and being down in time t; $D_i(t)=C_{i1}(t)\cup C_{i2}(t)\cup...\cup C_{im_i}(t)$ - set consist of at least one cut set $C_{ij}(t)$, which is down in time t, then Vesely-Fussell's measure is defined:

$$I^{VF}(i\mid t)=P\{D_i(t)\mid\Phi[\overline{X}(t)]=0\} \tag{10}$$

Vesely-Fussell's measure of importance can be interpreted as the probability that system failure state is caused by i-th element fail, when it is given that the system failure has occurred. For preliminary analysis can be use formula:

$$I^{VF}(i\mid t)\approx\frac{\sum_{j=1}^{m_i}\breve{Q}_j(t)}{Q_0(t)} \tag{11}$$

Where: $\breve{Q}_j(t)$ – unavailability of j-th cut set, which contains of i-th element, $Q_0(t)$ – unavailability of system.

The improvement potential reliability measure $I^{IP}(i|t)$ for i-th element is defined as the increase in system reliability if element i is replaced with a perfect component at time t. Improvement potential is interpreted as probability that i-th element is critically Cr for system and it fails in time t.

$$I^{IP}(i\mid t)=P\{Cr[\overline{X}(t),X_i=1]\cap[X_i(t)=0]\} \tag{12}$$

Critically measure of Lambert $I^{CR}(i|t)$ is given as probability that element i is critical for the system and is failed at time t, given that the system is failed at time t [9], what can be presented:

$$I^{CR}=P\{Cr[\overline{X}(t),X_i=1]\cap[X_i(t)=0]\mid\Phi[\overline{X}(t)]=0\} \tag{13}$$

$$I^{CR}=\frac{P\{Cr[\overline{X}(t),X_i=1]\cap[X_i(t)=0]\}}{P\{\Phi[\overline{X}(t)]=0\}} \tag{14}$$

Lambert’s measure can be connected with Birnbaum’s reliability measure of importance by means of formula:

$$I^{CR}(i \mid t) = \frac{I^{B}(i \mid t) \cdot q_i(t)}{Q_0(t)} \tag{15}$$

Birnbaum's measure of structural importance for i-th element is defined as the relative number of system states for which element i is critical for the system. Measure this can be presented by formula:

$$B_{\phi}(i) = \frac{\eta_{\phi}(i)}{2^{n-1}} \tag{16}$$

Where: $\eta_{\varphi}(i)$ is the total number of critical path vectors for i-th element.

A critical path vector for element i is a state vector of the other components in the system such that the system functions if and only if the i-th component functions. This measure is helpful for count the relative number of different states of the system (all other elements than i) which cause i-th element to be critical for the system. If all elements of system have unavailability q_i = 0,5, then $B_{\varphi}(i) = I^{B}(i|t_0)$.

3. Final conclusion

For finding elements, which dependability measures should be improved for incease of system reliability, the most useful are Birnbaum’s reliability importance measure and improvement potential.

For finding elements, which faults with highest probability will lead to system down, the most useful are Vesely-Fussell's and Lambert’s measures of importance. These two measures are useful for building of priority check lists and planed maintenance schedules. For rankings of elements which can be critical for system very useful is Birnbaum's measure of structural importance, which is time independent measure (depends only on system structure).

Based on comparison of different measures, conclude that all measures can be used for technical systems analysis. Proper measure should be selected accordingly to requirements of analysis and information about system. All importance measures can be supported by application of qualitative measures, for instance stream measure can be useful for finding elements, which are working more intensively then others in serial reliability structure of system.

Apart from measures shown in the paper, there are also many other measures (e.g. Natvig’s, Bergman’s, Barlow-Proshan’s), which have not shown here. Some of measures are evaluated based on function of system reliability, what can make them difficult for practically application [7, 10].

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